Propositional Logic

Source: http://www.slideshare.net/docfreeride/propositional-logic

Proposition (Statement)

- A statement is a declaratory sentence which is true or false but not both.

- In other words, a statement is a declarative sentence which has a definite truth table.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

Bill does not own a car.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

Bill does not own a car.

P=Bill owns a car.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

Bill does not own a car. ~P

P=Bill owns a car.

Using PL to translate natural language claims.

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Bill does not own a car. ~P

P=Bill owns a car.

It's not the case that Mary is short.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

Bill does not own a car. ~P

P=Bill owns a car.

It's not the case that Mary is short.

Q = Mary is short.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

Bill does not own a car. ~P

P=Bill owns a car.

It's not the case that Mary is short. ~Q

Q = Mary is short.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It’s not true that watching TV rots the brain.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It’s not true that watching TV rots the brain.

NOT:

P= It’s not true that watching TV rots the brain.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It's not true that watching TV rots the brain.

NOT:

P= It's not true that watching TV rots the brain.

P= Watching TV rots the brain.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It's not true that watching TV rots the brain. ~P

NOT:

P= It's not true that watching TV rots the brain.

P= Watching TV rots the brain.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It is immoral to torture cats.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It is immoral to torture cats.

NOT:

It’s not true it is moral to torture cats.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It is immoral to torture cats.

NOT:

It’s not true it is moral to torture cats.

P=It’s wrong to torture cats.

Using PL to translate natural language claims.

Negation: translate to reveal the logical structure of the negation in the statement.

It is immoral to torture cats. P

NOT:

It's not true it is moral to torture cats.

P=It's wrong to torture cats.

(*Some things are neither moral nor amoral*)

Conjunction:

Jenny went to the park and Bill went to the park.

Conjunction:

Jenny went to the park and Bill went to the park.

P=Jenny went to the park.

Q=Bill went to the park.

Conjunction:

Jenny went to the park and Bill went to the park.

P=Jenny went to the park.

Q=Bill went to the park. P•Q

Conjunction:

Jenny went to the park and Bill went to the park.

P=Jenny went to the park.

Q=Bill went to the park. P•Q

Jenny went to the park but Bill stayed home.

Conjunction:

Jenny went to the park and Bill went to the park.

P=Jenny went to the park.

Q=Bill went to the park. P•Q

Jenny went to the park but Bill stayed home.

P=Jenny went to the park.

R=Bill stayed home.

Some claims that look like conjunctions aren't:

The power was out for three days and the food in the fridge spoiled.

Some claims that look like conjunctions aren't:

The power was out for three days and the food in the fridge spoiled.

P=The power was out for three days.

Q=The food in the fridge spoiled.

Some claims that look like conjunctions aren't:

The power was out for three days and the food in the fridge spoiled.

P=The power was out for three days.

Q=The food in the fridge spoiled. P•Q

Some claims that look like conjunctions aren't:

The power was out for three days and the food in the fridge spoiled.

P=The power was out for three days.

Q=The food in the fridge spoiled. P•Q

Loses the temporal flow of the claim.

Some claims that look like conjunctions aren't:

The power was out for three days and the food in the fridge spoiled.

P=The power was out for three days.

Q=The food in the fridge spoiled. P•Q

Loses the temporal flow of the claim.

Not the same as: *The food in the fridge spoiled and the power was out for three days.*

Some claims that look like conjunctions aren't:

Fred and Jane made the soup.

Some claims that look like conjunctions aren’t:

Fred and Jane made the soup.

P=Fred made the soup.

Q=Jane made the soup. P•Q

Some claims that look like conjunctions aren't:

Fred and Jane made the soup.

P=Fred made the soup.

Q=Jane made the soup. P•Q

Misses the collective subject.

Some claims that look like conjunctions aren't:

Fred and Jane made the soup.

P=Fred made the soup.

Q=Jane made the soup. P•Q

Misses the collective subject.

Better: *Fred and Jane made the salad together.*

Some claims that look like conjunctions aren't:

Lentils have more protein than peanut butter and jelly.

Some claims that look like conjunctions aren't:

Lentils have more protein than peanut butter and jelly.

P=Lentils have more protein than peanut butter.

Q=Lentils have more protein than jelly. P•Q

Some claims that look like conjunctions aren't:

Lentils have more protein than peanut butter and jelly.

P=Lentils have more protein than peanut butter.

Q=Lentils have more protein than jelly. P•Q

Misses the additive comparison.

Some claims that look like conjunctions aren't:

Lentils have more protein than peanut butter and jelly.

P=Lentils have more protein than peanut butter.

Q=Lentils have more protein than jelly. P•Q

Misses the additive comparison.

Better: *Lentils have more protein than peanut butter and jelly combined.*

Disjunction:

Either my roommate will bring the textbook or my lab partner will let me borrow hers.

Disjunction:

Either my roommate will bring the textbook or my lab partner will let me borrow hers.

P=My roommate will bring the textbook.

Q=My lab partner will let me borrow her textbook.

Disjunction:

Either my roommate will bring the textbook or my lab partner will let me borrow hers.

P=My roommate will bring the textbook.

Q=My lab partner will let me borrow her textbook.

$P \vee Q$

Disjunction:

Either my roommate will bring the textbook or my lab partner will let me borrow hers.

P=My roommate will bring the textbook.

Q=My lab partner will let me borrow her textbook.

$P \vee Q$

"Inclusive or" – *true if either P or Q (or both) is true.*

Disjunction:

Either Meg Whitman or Jerry Brown will be the next Governor of California.

Disjunction:

Either Meg Whitman or Jerry Brown will be the next Governor of California.

P=Meg Whitman will be the next Governor of California.

Q=Jerry Brown will be the next Governor of California.

Disjunction:

Either Meg Whitman or Jerry Brown will be the next Governor of California.

P=Meg Whitman will be the next Governor of California.

Q=Jerry Brown will be the next Governor of California.

$P \vee Q$

Disjunction:

Either Meg Whitman or Jerry Brown will be the next Governor of California.

P=Meg Whitman will be the next Governor of California.

Q=Jerry Brown will be the next Governor of California.

$P \vee Q$ (*but note that only one of P and Q will actually be true!*)

Disjunction:

You can have either the soup or the salad with dinner.

Disjunction:

You can have either the soup or the salad with dinner.

P=You can have the soup with dinner.

Q=You can have the salad with dinner.

Disjunction:

You can have either the soup or the salad with dinner.

P=You can have the soup with dinner.

Q=You can have the salad with dinner.

“Exclusive or” – can’t have both!

Disjunction:

You can have either the soup or the salad with dinner.

P=You can have the soup with dinner.

Q=You can have the salad with dinner.

“Excusive or” – can’t have both!

$(P \vee Q) \bullet \sim(P \bullet Q)$

Disjunction:

Julie will be on time unless she sleeps through her alarm.

Disjunction:

Julie will be on time unless she sleeps through her alarm.

P=Julie will be on time.

Q=Julie will sleep through her alarm.

Disjunction:

Julie will be on time unless she sleeps through her alarm.

P=Julie will be on time.

Q=Julie will sleep through her alarm.

$P \vee Q$

The Syntax and Semantics of Propositional Logic

Valid arguments are truth-preserving

- If premises are true, conclusion **must** be true.
- Validity is a formal property – not a matter of content or context.

How to test whether an argument is valid?

Propositional Logic (aka Sentential Logic)

PL as a formal system to test arguments:

Step 1: Identify argument "in the wild" (in a natural language, like English)

Step 2: Translate the argument into PL

Step 3: Use formal test procedure within PL to determine whether argument is valid

Note that good translation is crucial!

Important features of PL

Symbols (to capture claims and logical connection between claims)

Syntax (the rules for how to take generate complex claims from simple ones)

Semantics (the meanings of the atomic units, and rules governing how meanings of atomic units are put together to form complex meanings)

Syntax of PL

Using logical connectives and operators (which connect or operate on propositions)

Symbols:

Use letters (P, Q, R, … X, Y, Z) to stand for specific statements

Unary propositional operator: ~

Binary propositional connectives: ⊃ , ≡ , • , ∨

Grouping symbols: ( ), [ ]

Syntax of PL

Negation; *not*: ~	~P
Conjunction; *and*: •	P•Q
Disjunction; *or*: ∨	P∨Q
Material conditional; *if … then ..*: ⊃	P⊃Q
Biconditional: *… if and only if …*: ≡	P≡Q

“Good grammar” in PL: well-formed formula (wff)

(1) Every statement letter P, … Z is a well-formed formula (wff)

(2) If p and q are wfs, then so are:

- (i) ~p
- (ii) (p•q)
    - (iii) (p∨q)
    - (iv) (p⊃q)
    - (v) (p≡q)

(3) Nothing is a wff unless rules (1) and (2) imply that it is.

Syntax of PL

Strings that are not wffs:

(P~Q)

(•QP)

(⊃R)

Strings that are wffs:

((P•Q)≡R)

~(X∨(Y⊃Z))

Syntax of PL

In PL, every compound formula is one of the following:

- negation
- conjunction
- disjunction
- conditional
- biconditional

To determine which one, isolate main connective or operator.

Syntax of PL

$(P \bullet Q)$ conjunction

$((P \bullet Q) \equiv R)$ biconditional

$(Y \supset Z)$ conditional

$(X \vee (Y \supset Z))$ disjunction

$\sim(X \vee (Y \supset Z))$ negation

Syntax of PL

By convention, we can drop the outermost set of parentheses if the main connective is not unary (~)

$(P \bullet Q) \equiv R$

$Y \supset Z$

$X \vee (Y \supset Z)$

$\sim(X \vee (Y \supset Z))$

Syntax of PL

Important note:

$\sim(P \bullet Q)$

is not equivalent to

$\sim P \bullet Q$

Semantics of PL

Semantic rules of PL tell us how the meaning of its constituent parts, and their mode of combination, determine the meaning of a compound statement.

Logical operators in PL determine what the truth-values of compound statements are depending on the truth-values of the formulae in the compound.

Semantics of PL

Logical operators defined by **truth-tables**.

(T= true, F=false)

Negation:

P	**~P**
T	F
F	T

Semantics of PL

Conjunction:

P	Q	P• Q
T	T	T
T	F	F
F	T	F
F	F	F

Semantics of PL

Disjunction:

P	Q	P∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Semantics of PL

Material conditional:

P	Q	P⊃ Q
T	T	T
T	F	F
F	T	T
F	F	T

Semantics of PL

Biconditional:

P	Q	P≡ Q
T	T	T
T	F	F
F	T	F
F	F	T

If, if only and if and only if

- **"Madison will eat the fruit if it is an apple."** (equivalent to **"Only if Madison will eat the fruit, is it an apple;"** or **"Madison will eat the fruit ← fruit is an apple"**)This states simply that Madison will eat fruits that are apples. It does not, however, exclude the possibility that Madison might also eat bananas or other types of fruit. All that is known for certain is that she will eat any and all apples that she happens upon. That the fruit is an apple is a *sufficient* condition for Madison to eat the fruit.
- **"Madison will eat the fruit only if it is an apple."** (equivalent to **"If Madison will eat the fruit, then it is an apple"** or **"Madison will eat the fruit → fruit is an apple"**)This states that the only fruit Madison will eat is an apple. It does not, however, exclude the possibility that Madison will refuse an apple if it is made available, in contrast with (1), which requires Madison to eat any available apple. In this case, that a given fruit is an apple is a *necessary* condition for Madison to be eating it. It is not a sufficient condition since Madison might not eat all the apples she is given.
- **"Madison will eat the fruit if and only if it is an apple"** (equivalent to "Madison will eat the fruit ↔ fruit is an apple")This statement makes it clear that Madison will eat all and only those fruits that are apples. She will not leave any apple uneaten, and she will not eat any other type of fruit. That a given fruit is an apple is both a *necessary* and a *sufficient* condition for Madison to eat the fruit.

"Madison will eat the fruit if it is an apple." (equivalent to **"Only if Madison will eat the fruit, is it an apple;"** or **"Madison will eat the fruit ← fruit is an apple"**)This states simply that Madison will eat fruits that are apples. It does not, however, exclude the possibility that Madison might also eat bananas or other types of fruit. All that is known for certain is that she will eat apples. That the fruit is an apple is a *sufficient* condition for Madison to eat the fruit.

- Imagine you know that Madison likes apples, and that you are in a party and someone asks you whether Madison will eat any fruit.